The Ex Ante Curse: Why is the Real Interest Rate so Confusing?

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Abstract

Conflicting conclusions arise in several studies of *ex ante* real interest rates, in particular the studies of Fisher equation and the real interest rate parity hypothesis. One possible explanation is that there are various techniques of computing the real interest rates and such differences in the methodology of constructing the series may lead to different conclusions in hypothesis testing. The source of the difficulty in the real interest rate measurement lies mainly on unobserved expected inflation which has to be estimated based on observed data and underlying assumptions of how people form their inflation forecasts. In this paper, we explore how researchers treat the expected inflation in the real interest rate calculation and statistical properties of different real interest rates.

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1 Introduction

The real interest rate is the key variable that affects saving and investment decisions, since it reflects the true costs of borrowing and the real returns from lending, adjusting for the inflation that is expected to occur over the period of time until maturity. Movements in the real interest rates are an important channel by which monetary shocks are transmitted to real economic activities.

Economic theory generally predicts that real rates of interest follow a stationary process, for example, the well-known Black-Scholes equation for option pricing relies on the constancy of the *ex ante* interest rate. However, many studies on the pattern of the real interest rate reveal mixed results. For instance, Fama (1975) found the real interest rate seems to be constant such that the nominal interest rate fully adjusts to changes in inflation, whereas Mishkin (1984), Huizinga and Mishkin (1984), and Hoffman and Schlagenhauf (1985) rejected the constancy of the real interest rate. Moreover, the consumption-based capital asset pricing model implies that time-series characteristics of the growth rate of consumption and the real interest rate should be the same. Since the growth rate of consumption is empirically a stationary process, the implication is that the real interest rate is as well. However, Rose (1988) found strong evidence that real interest rates are nonstationary, even though the growth rate of consumption appear to be stationary.

There are two main strands of empirical studies that focus extensively on the use of the real rate of interest: the Fisher hypothesis and the Real Interest Parity (RIRP) hypothesis. The Fisher relation indicates that the nominal interest rate adjusts fully to changes in the expected rate of inflation such that there is one-to-one relationship between them and that the expected real rate of returns remains constant with respect to changes in expected inflation. For the RIRP hypothesis, given Fisher relations, the Uncovered Interest Parity condition, and the *ex ante* version of Purchasing Power Parity are satisfied for each country, the *ex ante* real interest rates are equalized across countries. Researchers employ a wide variety of approaches of measuring the expected real returns on assets when attempting to test these hypotheses. These methodologies differ in how to treat the expected inflation in the real interest rate calculation as well as what proxies to use for the percentage change in price levels.

In this paper we examine various methodologies used in the literature and compare the differences in time-series properties of these real interest rates. We do not attempt to identify the “correct” specification. Instead, we attempt to determine whether different methods yield different real interest rate series. Particularly, we aim to investigate how robust the stationarity of real interest rate is to the underlying approach of deriving the rates.

We select six methodologies of constructing the *ex ante* real interest rates, (*i*) the *ex post* real interest rate, (*ii*) AR(4) inflation forecast, (*iii*) Mishkin’s linear projection, (*iv*) rolling regression, (*v*) recursive least squares, and (*vi*) regime-switching techniques. The methods (*ii*)-(*v*) can be viewed as the linear regression approaches, since the estimations are based on the linear regression model with different specifications and the included variables. For example, Mishkin’s approach is the extended version of the AR(4) model, of which more macroeconomic variables are added into the linear regression model of the past inflation rate. On the other hand, the regime-switching method estimates nonlinearly the pattern of the real interest rate after including possible regime shifts constructed by a Markov-chain probability.[1].

Furthermore, each inflation rate can be calculated using either the period-to-period annualized inflation rate or the inflation rate calculated as compared to last year. Thus a total of 24 different real

[1] Under the assumption of rational expectations, we are able to estimate the *ex ante* real rates using approach (*i*)-(*vi*)

interest rates are calculated.[2] These 24 series are then compared to examine the time series properties of each series. We compare their distributions by using a normality test and the Augmented Dickey-Fuller unit root test to investigate whether the real interest rates from different approaches yield different results regarding stationarity. Implicitly, the past literature has assumed that it does not matter which type of real interest rate to use. If that is the case, all of the 24 real interest rates should have similar time series properties. If that is not the case, then this study may shed some light on which type of assumptions lead to similar times series processes, and whether any type of real interest rate calculation is very different from the others.

Our results clearly indicate that the approaches used in constructing real interest rates matter. The time series properties of the various constructed real interest rates seem to be sensitive to the approach used, the frequency of the data, and the method of inflation rate calculation. Although the U.S *ex ante* real interest rates from different approaches move in a similar pattern throughout the sample period, they seem to follow different time series processes.

The following section discusses the construction of real interest rates from different econometric methodologies. The third section discusses the data used to generate the real interest rates. The fourth section discusses the results, and the last section provides some conclusions and further research.

2 Constructing Real Interest Rates

Based on the Fisher equation, the nominal rate of interest can be thought of as the equilibrium expected real return plus the market's assessment of the expected rate of inflation. Thus, the *ex ante* real interest rate, $r^e_{t,t+1}$, is defined as:

$$r^e_{t,t+1} = i_t - \pi^e_{t+1} \tag{1}$$

where

[2] There are 6 methods of the real interest rate construction, using 4 different inflation rate series. There are two common approaches for calculating the annual rate of inflation. First, most researchers construct the inflation rate by obtaining the period-to-period changes in the logarithm of price and then annualize the series; that is, for monthly data, the inflation rate is

$$\pi_t = \ln\left(\frac{P_t}{P_{t-1}}\right)^{12}$$

and the quarterly annualized inflation is defined as

$$\pi_t = \ln\left(\frac{P_t}{P_{t-1}}\right)^{4}$$

Alternatively, the annual inflation rate can be constructed as the following:

$$\pi_t = \ln\left(\frac{P_t}{P_{t-12}}\right)$$

$$\pi_t = \ln\left(\frac{P_t}{P_{t-4}}\right)$$

for monthly and quarterly data, respectively. This year-to-year inflation rate calculation tends to yield a slightly smoother inflation process, since it avoids discreteness of reported CPI data. CPI is reported month-to-month or quarter-to-quarter in terms of discrete numbers with small changes from one period to the another. Using a period-to-period inflation rate will magnify the effect of price changes by the exponential of 12 for monthly or 4 for quarterly data. Thus, the obtained inflation rate will fluctuate dramatically.

i_t is the nominal interest rate from holding the one-period bond from t to $t+1$,
$r^e_{t,t+1}$ is the one-period real rate of interest for the bond maturing at time $t+1$, expected at time t; and
π^e_{t+1} is the rate of inflation from t to $t+1$, expected by the agents in the market at time t.

Many authors have tried to mimic how agents form their expectations about inflation rate using a wide range of models from simple Autoregressive models to elaborate general equilibrium models. Consequently, there is very little agreement among researchers on how to construct an *ex ante* real interest rate, and that lack of agreement might lead to very different results in the time series properties of the constructed real rates. For instance, Mishkin (1984) derived the *ex ante* real rate using the *ex post* linear projection and rejected the constancy of the real interest rate; whereas, Garcia and Perron (1996) using a three-state Markov-switching model found that the real rate is constant with occasional shifts. These different models often lead to different assumptions about the time series properties of real interest rates, and thus alter the type of methodology that is used for testing purposes. For example, many researchers have found the real rate to be nonstationary, resulting in the use of cointegration type methods (see Crowder and Hoffman (1996)). Many different conclusions have been made about subjects such as: real interest rate equalization, capital flows, and contagion effects, perhaps due to the type of method used to calculate the real interest rates. This section provides a brief summary of the approaches researchers have taken in estimating the *ex ante* real rate of interest and expected inflation.

2.1 Pure Rational Expectations

Studying real interest rates seems to be problematic in the sense that an *ex ante* real interest rate is unobservable. Thus, many studies have to develop a method to estimate the *ex ante* real interest rate and then impose some structure and assumptions into models. The simplest assumption is to assume perfect rational expectations, and thus the *ex post* rate is the best prediction of the *ex ante* real rate with a zero mean error term. Using realized inflation rate during period $t+1$, π_{t+1}, one may compute *ex post* real returns from the one-period bond as:

$$r^p_{t,t+1} = i_t - \pi_{t+1} \tag{2}$$

By assuming rational expectations, π^e_{t+1} is the mathematical expectation of π_{t+1} conditional on all the relevant information available to the agents at time t. Let ϕ_t be the set of all available information at the time inflation expectations are formed, then we have

$$\pi^e_{t+1} = E_t(\pi_{t+1}|\phi_t) \tag{3}$$

and hence

$$\pi_{t+1} - \pi^e_{t+1} = \varepsilon_{t+1} \tag{4}$$

where ε_{t+1} is the inflation forecast error with zero mean and, by construction, is uncorrelated with ϕ_t. Thus the rational expectations hypothesis implies

$$r^p_t \equiv i_t - \pi_{t+1} \equiv r^e_t - (\pi_{t+1} - \pi^e_{t+1}) \equiv r^e_t - \varepsilon_{t+1} \tag{5}$$

such that the *ex ante* real rate equals the *ex post* real rate and the inflation forecast errors.

2.2 Time Series Forecasting Models

To quantify the unobserved component of the real interest rate, time series models can be useful in approximating the expected rate of future inflation using only the past behavior of the realized inflation rate, which is readily available. The types of time series models that have been used in prior researches of the expected inflation are: ARMA model, Mishkin's linear projection technique, the recursive least squares method, the rolling regression, and the Markov-switching model.

2.2.1 ARMA Model

Autoregressive representations are appealing to researchers because, for forecasting purposes, they link the present observable data to the past history of the data so that we can extrapolate to form a forecast of future observable data based on present and past observations. Thus, we can use the AR(p) representation to express the current observable inflation rate as a function of past realized rate of inflation:

$$\pi_t = \varphi_1\pi_{t-1} + \varphi_2\pi_{t-2} + \dots + \varphi_p\pi_{t-p} + \varepsilon_t \tag{6}$$
$$\varepsilon_t \sim WN(0, \sigma^2)$$

The expected inflation rate can be obtained by fitting the AR representations on the actual inflation rate and forming a one-period ahead forecast from the estimated AR coefficients as follows

$$\pi^e_{t+1} \equiv \hat{\pi}_{t+1} = \hat{\varphi}_1\pi_{t-1} + \hat{\varphi}_2\pi_{t-2} + \dots + \hat{\varphi}_p\pi_{t-p} \tag{7}$$

where $\{\hat{\varphi}_1, \hat{\varphi}_2, \dots, \hat{\varphi}_p\}$ are the estimated coefficients of $\{\varphi_1, \varphi_2, \dots, \varphi_p\}$. An example of empirical study that applies this approach is Fountas and Wu (1999). They construct the *ex ante* real interest rate by creating an expected inflation series from a 4-period moving average of actual inflation rates.

2.2.2 Mishkin's Linear Projection Technique

Mishkin (1984), Cumby and Mishkin (1986), Huizinga and Mishkin (1984) extend the ARMA model of expected inflation by adding relevant macroeconomic variables, hereafter referred as Mishkin's approach. Mishkin's linear projection technique is based on the rationality of inflation forecasts, which implies that the *ex ante* real rate equals the expected real return on the one-period bond, conditional on available information at time t; that is,

$$r^e_{t,t+1} = E(r^p_{t,t+1}|\phi_t) \tag{8}$$

Since the *ex ante* real rate is unobservable, Mishkin used a set of observable variables, X_t, which are elements of the available information set ϕ_t, to linearly project $r^e_{t,t+1}$ into X_t as

$$P(r^e_{t,t+1}|X_t) = X_t\beta \tag{9}$$

and then

$$r^e_{t,t+1} = X_t\beta + u_t \tag{10}$$

where $u_t = r^e_{t,t+1} - P(r^e_{t,t+1}|X_t)$ is the projection errors and u_t is orthogonal to X_t. Mishkin's choice of X_t includes four lags of the inflation rate, one lag of money growth (M1), the nominal eurodollar interest rate and the fourth order polynomial in time (since his data is quarterly).

2.2.3 Recursive Least Squares

The recursive regression is the least-square estimation method that keeps the beginning point of the sample the same and only moves the sample end points. Start with the standard autoregressive moving-average $ARMA(p,q)$ model

$$\Phi(L)\pi_t = \Theta(L)\varepsilon_t \tag{11}$$

$$\varepsilon_t \sim WN(0,\sigma^2)$$

where $t = 1,...,k-1,k,k+1,...,T$, $\Phi(L) = 1-\phi_1 L-\phi_2 L^2 - ... - \phi_p L^p$ and $\Theta(L) = 1+\theta_1 L+\theta_2 L^2 + ... + \theta_q L^q$. Instead of immediately using all data, $t = 1,...,T$, we begin with a small subset, that is, we begin with the first k observations and estimate the coefficients of the model with the maximum log-likelihood estimation. Then, we update our estimates by using the first $k+1$ observations, and so on, until the sample is exhausted. At the end, we have a set of recursive parameter estimates $\hat{\Phi}_t$ and $\hat{\Theta}_t$, $t = k,...,T$. In this particular case, agents start to form their forecasts after obtain some information about the inflation rate. With the passage of time, they have never forgotten the old information and collect more and more new information, which allows them to make estimates that are more precise as sample size grows.

2.2.4 Rolling Regression

Juntilla (2001) uses the rolling regression technique to approximate the inflation forecasts in the existence of exogenous regime shifts in the inflation rate. The purpose of these estimations is to take into account of the possibility that agents, basing their expectations of future inflation on the past behavior of actual inflation, would be able to learn from the changes in the most recent observations on inflation. The key idea is that at a certain point in time there is a change in the inflation process, which might be caused by an exogenous event or a change in policy regime. However, after the change has occurred, agents may require time to learn about the new process using the knowledge from the past.

The rolling regression technique requires a fixed sample size, by changing both the starting and the end point of the observation to keep the sample size unchanged. The agents are not concerned about the information that is too old but, instead, they use information about the past actual inflation rate in the fixed time interval moving window to form their forecasts. That is, for example, agents use a 5-year range of data on inflation to predict the next period inflation. The first iteration computes coefficient estimates of $\hat{\Phi}_t$ and $\hat{\Theta}_t$ for an ARMA(p,q) model as described in the equation (11) for a 5-year moving window starting with the first observation until the 60th observation (monthly data). Then, the second iteration will be based on information from the second observation to the 61st observation and so on. In each iteration, the sample size remains the same at 60 observations. As agents move along the time line, they learn about the pattern of inflation rate in the past five years and then predict the future rate of inflation. The estimated coefficients reflect the impact of new information on the markets. If the coefficients display significant time variation when the subsample is "rolled over", this is a strong indication of instability.

2.2.5 Regime-Switching Model

Researchers have found evidence that supports distinct switches in the inflation regimes in the past. For example, Huizinga and Mishkin (1984) find that a significant shift in the stochastic process of real rates occur sometime around October 1979 when the Fed changes its policy procedure. These switches would affect rational agents' forecasts and the degree on uncertainty associated with future inflation. Uncertainty about the inflation process leads agents to make forecasts that appear systematically biased and in some sense "irrational". Ignoring this can lead to incorrect conclusions about the behavior of the *ex ante* real rate of returns.

Garcia and Perron (1996) considered the behavior of the U.S. real interest rate, using the methodology of Hamilton (1989), by allowing three possible regimes affecting both the mean and the variance. They were motivated by recent empirical results of potential nonstationarity of the *ex ante* real interest rates, which is an important issue for public policy and has theoretical implications. Their empirical analysis uses the *ex post* real interest rate, under the assumption that agents use available information efficiently to analyze the *ex ante* real rate. Their approach allows nonstationarity in the form of infrequent changes in mean and variance caused by important structural events. Various specification tests used by Garcia and Perron suggest that the real interest rate may not have a unit root. Instead, the real rate tends to be mean reverting within each regime.

The three-state Markov-switching mean-variance model explicitly accounts for regime shifts in an autoregressive model of the *ex post* real rate as follows:

$$(y_t - \mu_{s_t}) = \phi_1(y_{t-1} - \mu_{s_{t-1}}) + \phi_2(y_{t-2} - \mu_{s_{t-2}}) + e_t, \tag{12}$$

where y_t is an AR(2) process of the *ex post* real interest rate calculated by subtracting the inflation rate from the nominal interest rate and μ_{s_t} and $\sigma^2_{s_t}$ is the mean and variance switching parameters when state S_{jt} is realized for $j = 1,2,3$, respectively.

$$e_t \sim N(o, \sigma^2_{s_t}), \tag{13}$$

$$\mu_{s_t} = \mu_1 S_{1t} + \mu_2 S_{2t} + \mu_3 S_{3t}, \tag{14}$$

$$\sigma^2_{s_t} = \sigma^2_1 S_{1t} + \sigma^2_2 S_{2t} + \sigma^2_3 S_{3t}, \tag{15}$$

$$S_{jt} = 1, \text{ if } S_t = j \text{ and } S_{jt} = 0, \text{ otherwise } j = 1,2,3, \tag{16}$$

The state variable S_t, $t = 1,2,...,T$, is unknown a priori, that is, the dates of structural breaks are unobservable. Therefore, to determine the log likelihood function, we need to consider the joint density of y_t and the unobserved S_t variable and then integrate the S_t variable out of the joint density to obtain the marginal density of y_t. Without a priori assumptions about the stochastic behavior of S_t, however, this will not be possible. We make the assumption of the evolution of S_t to be a first-order Markov-switching process.

$$p_{ij} = Pr[S_t = j | S_{t-1} = i], \ \sum_{j=1}^{3} p_{ij} = 1, \tag{17}$$

Under the assumption of rational expectations, given the probabilities and parameter estimates, the *ex ante* real interest rate can be estimated in the following way:

$$E(y_t|\tilde{y}_{t-1}) = E(\mu_{S_t}|\tilde{y}_t) + \phi_1 E(y_{t-1} - \mu_{s_{t-1}}|\tilde{y}_t) + \phi_2 E(y_{t-2} - \mu_{s_{t-2}}|\tilde{y}_t) \tag{18}$$

where $\tilde{y}_t = [y_1 ... y_t]$.[3]

[3]We follow Kim and Nelson (1999) in applying the Gibbs-sampling methods to the three-state Markov-switching model and obtaining distributions and parameter estimates. See Casella and George (1992) for examples and applications of Gibbs-sampling algorithm.

3 Data

We divide our empirical study into two time frequencies: monthly and quarterly. We attempt to investigate whether the choice of data frequency affects the properties of the derived real interest rates. For monthly data (1971:1-2003:12), we use the eurodollar interest rate available from the Federal Reserve Bank at St. Louis for the nominal interest rate and we employ the seasonally adjusted CPI series to calculate annualized inflation. For quarterly data (1971:I-2003:IV), the 3-month Treasury bill rate (secondary market) is used for the nominal interest rate and the annual inflation rate is computed from a quarterly seasonally adjusted CPI series.

Beside the choice of data frequency, we explore if the computations of the inflation rate affect the dynamics of the obtained inflation series and hence the constructed real interest rates. We use both monthly and quarterly CPI to obtain the rate of inflation from two common approaches: the period-to-period calculation and the year-to-year calculation. Only a few studies indicate how they compute the rate of inflation, of which the majority of them use the period-to-period approach to calculate the inflation rates. For example, in Chen (2001) study of a model for real interest rate, the quarter-to-quarter annualized inflation was used. However, Gagnon and Unferth (1995) and Fountas and Wu (1999) use the year-to-year approach to calculate monthly and quarterly annualized rate of inflation, respectively. Since many authors do not describe explicitly how they calculated the inflation rate, we show all the possible ways economists may calculate the inflation rate to determine the sensitivity of the obtained real interest rate to the methods of calculation. If that is the case, then the different ways of obtaining the expected inflation rate could result in different results of the tests of the Fisher equation and real interest rate parity.

4 Results

The *ex ante* real interest rates are computed using six different methods. These methods are (*i*) the *ex post* real interest rate, (*ii*) an AR(4) inflation forecast, (*iii*) Mishkin's linear projection, (*iv*) rolling regression, (*v*) recursive least squares, and (*vi*) Markov-switching technique. [4]

In additional to examine descriptive statistics of each constructed real interest rate, we investigate its stationarity using the augmented Dickey-Fuller (ADF) test. The ADF test is carried out by estimating the time series with serial correlation in errors described as

$$\Delta y_t = a + \alpha y_{t-1} + \sum_{j=1}^{k} \beta_j \Delta y_{t-j} + \varepsilon_t \tag{19}$$

[4]To construct the *ex ante* real interest rate from method (*iv*) and (*v*), we first fit the inflation rate series with the autoregressive moving-average models. The choice of ARMA specification of the inflation rate is based on Box-Jenkins model selection method. We also assume that the specifications of the ARMA model remain unchanged through time. Only the estimated coefficients are allowed to vary and roll over. and then compute the one-period ahead forecasts, using rolling and recursive regression, in order to derive the expected rate of inflation. In the regime-switching model (*vi*), we employ the AR(2) process to the *ex post* real interest rate with three-state Markov-switching mean and variance. The *ex ante* real interest rate will be estimated from all available information of the *ex post* rate in a combination with the possible state shifts in the mean and the variance. Instead of using the maximum likelihood estimate, we use the Gibbs-sampling technique. We implemented the Gibbs-sampling technique using the GAUSS program available from Nelson's web page at http://weber.u.washington.edu/c̃nelson/SSMARKOV.html. We discard the first 1,000 draws of Gibbs-sampling and the analysis is based on the next 9,000 draws.

where $y_t = a + \rho y_{t-1} + \varepsilon_t$, $\varepsilon_t = \phi\varepsilon_{t-1} + e_t + \theta e_{t-1}$, $\alpha = \rho - 1$, and $t = 1, ..., T$. The augmented terms Δy_t of higher order lags are included into equation (19) to correct the serial correlations of the disturbances ε_t. The number of k lags are selected by Schwarz information criteria (SIC). The null hypothesis of a unit root ($\alpha = 0$) is tested against the alternative hypothesis of stationarity ($\alpha < 0$). The test statistic is evaluated using the conventional t-ratio for α and the critical value is obtained by MacKinnon's updated version of Dickey-Fuller critical values[5].

4.1 Real Interest Rates

In order to construct each of the real interest rate series, we use the data of nominal interest rate and compute the inflation rates for the entire sample period of 1971:1 to 2003:12 for monthly and 1971:I to 2003:IV for quarterly frequency. For comparison purposes, we limit our analysis of time series properties of the constructed real rates, to the common sample period of 1975:1 (1975:I) to 2002:12 (2002:IV) for monthly (quarterly) data.[6]

Tables 1-4 list the descriptive statistics of the real short-term interest rats as well as the unit toot test statistics. The critical value the unit root test is reported beneath the tables. Rejection of the null hypothesis at 0.05 significant level is marked by one asterisk. Table 5 reports the results of the mean and variance equality testing for both monthly and quarterly data. We employ the analysis of variance (ANOVA) to examine whether different approaches of constructing *ex ante* real interest rates would provide the series with equal mean. For the variance equality testing, a Brown-Forsythe test is used to evaluate the null hypothesis that the variance in all series are equal against the alternative that at least one series has a different variance.

4.1.1 Monthly Data

Tables 1 and 2 report the statistics of the *ex ante* real interest rates that we have derived from each methodology, using monthly data with the month-to-month and the year-to-year inflation rates, respectively. Clearly, the *ex ante* real interest rates that we derived from all the methods with the month-to-month inflation rate yield a similar mean and median of the series. This finding is confirmed by the probability of 0.98 of accepting the mean equality among all the real interest rates, as shown in the first column of table 5. The average of the *ex ante* real interest rates is 2.76 percent. Mishkin's real interest rate seems to be least variable with 2.1 standard deviations, while the *ex post* real interest rate is the most volatile series with 3.4 standard deviations. The variance equality test clearly suggests that at least one variance of these series are significantly different from one another. All series are slightly positively skewed and have small leptokurtosis. Using the Jarque-Bera normality test, we reject the normality in all the series.

Similarly, the averages of the *ex ante* real interest rates are also approximately 2.86 percent when applying the year-to-year annualized inflation rate to the construction of the real interest rates. The second column of table 5 also show that we are failed to reject the equality among the means of the *ex ante* real interest rates. However, the variances are significantly different from each other. The rolling regression and recursive least squares real rates seem to be the most variable series

[5] see Dickey and Fuller (1981) and MacKinnon (1996).

[6] Since the rolling regression and the recursive least squares techniques require a starting period, we lose the first five years of data. Consequently, the real interest rate series begin in the first period of the year 1975. Moreover, we lose the last year observations in the procedure of computing the year-to-year inflation rate.

with the standard deviations of 3.4, while the Mishkin's real interest rate yields the lowest standard deviations of 2.3. All of the series seem to be slightly skewed with a long right tail. The *ex post* and the real interest rates obtained by using the AR(4), Mishkin's, and regime-switching approaches appear to be a normal distribution as indicated by Jarque-Bera statistics and the kurtosis of 3. The *ex ante* real rates of interest from the rolling and recursive regressions suffer from leptokurtosis. The results in the unit root property of all real interest rates are mixed.

The unit root tests in tables 1 and 2 indicate that overall the real interest rates appear to be sometimes stationary and at other times the rates appear to be nonstationary. For example, the rolling regression and recursive least squares real interest rates are more likely to be stationary than the other rates. Interestingly, the ex post, Mishkin, and regime-switching real interest rates are unit root series in one type of inflation rate calculation and become stationary process in the other type of inflation calculation. This results indicate that, for some real interest rates, the choice of inflation calculation matters.

4.1.2 Quarterly Data

The average real returns on the 3-month Treasury bill are slightly lower than the average of real returns on the 1-month eurodollar, as shown in table 3. The mean and median of the real interest rates in all approaches are similar. The test of mean equality fails to reject the null hypothesis of equal means in each series. The *ex post* real rate appears to have the highest standard deviations of 2.8 and Mishkin's approach yield the least variable real rate. The rejection of variance equality supports this observation that at least one of the series have significantly different variance. Unlike what we have observed in the monthly data, all series are distributed normally with kurtosis around 3.0 and skewness are close to zero. The Jarque-Bera statistics indicates that the normality hypothesis cannot be rejected in contrast to the findings in tables 1 and 2. However, the results in the unit root testing for the quarter-to-quarter case are surprising. We find that the real interest rates are $I(1)$ in all cases under the conventional ADF unit root test.

When we use the quarterly year-to-year inflation rate to construct the real interest rates, we obtain series that yield similar mean and median, and the means of these series are tested to be equal. Mishkin's approach provides a less variable real interest rate while the rolling regression yields the most volatile series. All of the series appear to have a normal distribution, except for Mishkin's real interest rate series which suffers from leptokurtosis. The results from the unit root test indicates that the real interest rate from the recursive least squares, rolling regression, and AR(4) approaches are $I(0)$. The *ex post* real rate seems to be nonstationary as well as the real interest rate from regime-switching and Mishkin models. The unit root tests in tables 3 and 4 indicate a similar conclusion to the monthly rates such that the overall real interest rates appear to be sometimes stationary and at other times nonstationary depending on the constructing methods and the inflation calculation methods.

5 Conclusions

Since the time series properties of the expected real interest rate are important in many economic theories, we review the different approaches of constructing the *ex ante* real interest rate that have been used in prior literature. We construct the *ex ante* real interest rate from the following methods: (*i*) the *ex post* real interest rate, (*ii*) the AR(4) inflation forecasts, (*iii*) Mishkin's linear projection, (*iv*) the rolling regression, (*v*) the recursive least squares, and (*vi*) the regime-switching techniques. We then examine the time series properties of these real interest series.

Our findings indicate that the obtained real interest rates from different approaches yield different time series processes, although they appear to have the same mean and vary across time in similar patterns over the sample period. Particularly, the inconsistent results obtained by previous authors concerning the stationarity of the real interest rate appears to depend on the method of constructing the real rate, the sample frequency, and the computations of the inflation rate. Therefore, we would anticipate that hypothesis testing that involves the *ex ante* real interest rate will provide a wide range of conclusions as a result of various constructing methods and the choice of data used to construct the real rates.

References

[1] Casella, George and George, Edward (1992) “Explaining the Gibbs Sampler.” *American Statistician*, 46(3), 167-174.

[2] Chen, Li-Hsueh (2001) “A model for ex ante real interest rates.” *Applied Economics Letters*, 8, 713-718.

[3] Crowder, W. and Hoffman, D. (1996) “The long-run relationship between nominal interest rates and inflation: the Fisher equation revisited.” *Journal of Money, Credit and Banking*, 28(1), 102-118.

[4] Cumby, Robert and Mishkin, Frederic (1986) “The international linkage of real interest rates: the European-US connection.” *Journal of International Money and Finance*, 5, 5-23.

[5] Dickey, D. and Fuller, W. (1981) “Likelihood ratio statistics for autoregressive time series with a unit root.” *Econometrica*, 49, 1057-1072.

[6] Fama, Eugene (1975) “Short-tern interest rates as predictors of inflation.” *American Economic Review,* 65(3), 269-282.

[7] Fountas, Stilianos, and Wu, Jyh-lin (1999) “Testing for real interest rate convergence in European countries.” *Scottish Journal of Political Economy*, 46(2), 158-174.

[8] Garcia, Rene, and Perron, Pierre (1996) “An analysis of real interest under regime shift.” *Review of Economics and Statistics*, 78, 111-125.

[9] Hamilton, James (1989) “A new approach to the economic analysis of nonstationary time series and the business cycle.” *Econometrica*, 57(2), 357-384.

[10] Hoffman, Dennis and Schlagenhauf, Don (1985) “Real interest rates, anticipated inflation, and unanticipated money: a multi-country study.” *Review of Economics and Statistics*, 67(2),284-296.

[11] Huizinga, John and Mishkin, Frederic (1984) “Inflation and real interest rates on assets with different risk characteristics.” *Journal of Finance*, 39(3) , 699-712.

[12] Juntilla, J. (2001) “Structural breaks ARIMA model and Finnish infltion forecasts.” *International Journal of Forecasting*, 17, 203-230.

[13] Kim, Chang-Jin and Nelson, Charles (1999) *State-Space Models with Regime Switching: Classical and Gibbs-Sampling Approaches with Applications.* The MIT Press, Massachusetts.

[14] MacKinnon, James (1996) “Numerical distribution functions for unit root and cointegration tests.” *Journal of Applied Econometrics*, 11, 601-618.

[15] Mishkin, Frederic (1984) “Are real interest rates equal across countries? An empirical investigation of international parity conditions.” *Journal of Finance*, 39(5), 1345-1357.

[16] Rose, Andrew (1988) “Is the real interest rate stable?” *Journal of Finance*, 43(5), 1095-1112.

Table 1: Monthly *ex ante* real interest rates using $\ln(\frac{p_t}{p_{t-1}})^{12}$ inflation rate (1975:12-2002:12)

Statistics	Ex Post	AR(4)	Mishkin	Rolling Reg.	RLS	Regime-Switching
Mean	2.904	2.845	2.757	2.859	2.923	2.898
Median	2.938	2.661	2.690	2.676	2.779	2.616
Maximum	15.520	12.448	10.320	14.425	13.434	10.129
Minimum	-7.333	-3.391	-1.742	-4.320	-3.256	-2.171
Standard Deviations	3.398	2.718	2.111	2.955	2.753	2.381
Skewness	0.572	0.795	0.823	0.524	0.364	0.617
Kurtosis	3.810	3.859	4.089	3.919	3.486	2.999
Jarque-Bera	26.627	44.208	52.767	26.327	10.355	20.652
Probability	(0.001)	(0.001)	(0.001)	(0.001)	(0.006)	(0.001)
ADF test:	-3.216*	-1.787	-1.243	-2.301*	-2.331*	-1.408
Lagged terms	2	4	6	3	2	3

RLS stands for the Recursive Least Squares model. The ADF test statistics are reported for the test without a drift term. The number of augmented terms in the ADF unit root test is based on SIC. * indicates rejection of the null hypothesis at a 5% significance level. Critical values for the ADF test is -1.942.

Table 2: Monthly *ex ante* real interest rates using $\ln(\frac{p_t}{p_{t-12}})$ inflation rate (1975:12-2002:12)

Statistics	Ex Post	AR(4)	Mishkin	Rolling Reg.	RLS	Regime-Switching
Mean	2.823	2.815	2.771	2.968	2.955	2.794
Median	2.831	2.890	2.875	2.885	2.838	2.820
Maximum	9.715	9.815	9.796	13.259	13.226	10.209
Minimum	-3.075	-3.928	-3.815	-3.729	-3.703	-3.545
Standard Deviations	2.404	2.387	2.337	3.365	3.361	2.403
Skewness	0.330	0.316	0.336	0.674	0.686	0.342
Kurtosis	2.889	2.995	3.221	3.794	3.792	3.025
Jarque-Bera	6.071	5.407	6.783	33.017	33.966	6.326
Probability	(0.048)	(0.067)	(0.034)	(0.001)	(0.001)	(0.042)
ADF test:	-1.893	-1.924	-2.111*	-2.316*	-2.348*	-1.952*
Lagged terms	2	2	2	1	1	2

RLS stands for the Recursive Least Squares model. The ADF test statistics are reported for the test without a drift term. The number of augmented terms in the ADF unit root test is based on SIC. * indicates rejection of the null hypothesis at a 5% significance level. Critical values for the ADF test is -1.942.

Table 3: Quarterly *ex ante* real interest rates using $\ln(\frac{p_t}{p_{t-1}})^4$ inflation rate (1975:IV-2002:IV)

Statistics	Ex Post	AR(4)	Mishkin	Rolling Reg.	RLS	Regime-Switching
Mean	2.079	2.023	1.925	2.211	2.032	2.010
Median	2.414	1.931	2.168	2.346	2.253	2.040
Maximum	10.426	6.886	7.806	9.079	7.957	6.697
Minimum	-4.268	-4.945	-4.583	-4.151	-4.108	-2.049
Standard deviations	2.886	2.158	1.989	2.640	2.533	2.002
Skewness	0.215	-0.009	0.084	0.120	-0.122	-0.078
Kurtosis	3.355	3.090	3.614	3.133	3.006	2.795
Jarque-Bera	1.410	0.038	1.839	0.341	0.271	0.303
Probability	(0.494)	(0.981)	(0.399)	(0.843)	(0.873)	(0.859)
ADF test:	-1.531	-1.487	-1.675	-1.688	-1.734	-1.531
Lagged terms	2	2	2	3	3	3

RLS stands for the Recursive Least Squares model. The ADF test statistics are reported for the test without a drift term. The number of augmented terms in the ADF unit root test is based on SIC. * indicates rejection of the null hypothesis at a 5% significance level. Critical values for the ADF test is -1.942.

Table 4: Quarterly *ex ante* real interest rates using $\ln(\frac{p_t}{p_{t-4}})$ inflation rate (1975:IV-2002:IV)

Statistics	Ex Post	AR(4)	Mishkin	Rolling Reg.	RLS	Regime-Switching
Mean	2.029	2.026	1.983	2.200	2.173	1.909
Median	2.159	2.169	2.071	2.348	2.262	2.009
Maximum	7.647	6.839	7.331	11.691	11.047	6.955
Minimum	-4.947	-6.118	-5.992	-5.833	-5.140	-4.197
Standard deviations	2.259	2.182	2.004	3.292	3.188	2.146
Skewness	-0.169	-0.200	-0.161	0.128	0.164	-0.154
Kurtosis	2.963	3.667	4.612	3.602	3.364	2.703
Jarque-Bera	0.524	2.741	12.267	1.945	1.089	0.834
Probability	(0.769)	(0.254)	(0.002)	(0.378)	(0.580)	(0.659)
ADF test:	-1.289	-2.389*	-1.502	-2.414*	-2.458*	-1.566
Lagged terms	2	0	4	0	0	3

RLS stands for the Recursive Least Squares model. The ADF test statistics are reported for the test without a drift term. The number of augmented terms in the ADF unit root test is based on SIC. * indicates rejection of the null hypothesis at a 5% significance level. Critical values for the ADF test is -1.942.

Table 5: Tests for Equality of Means and Variance between Real Interest Rate Series (1975:12-2002:12)

	Real interest rates from different approaches			
	Monthly		Quarterly	
	$\ln(\frac{p_t}{p_{t-1}})^{12}$	$\ln(\frac{p_t}{p_{t-12}})$	$\ln(\frac{p_t}{p_{t-1}})^4$	$\ln(\frac{p_t}{p_{t-4}})$
Mean Equality Test:				
Test Statistics	0.152	0.294	0.171	0.209
	(0.980)	(0.294)	(0.973)	(0.959)
Variance Equality Test:				
Test Statistics	13.238	9.617	3.516	6.456
	(0.000)	(0.000)	(0.000)	(0.000)

The mean equality test is based on ANOVA. The variance equality test is based on Brown-Forsythe test. The reported test statistics are the F-statistics follow F-distribution with (5,1937) and (5,648) degrees of freedom for monthly and quarterly data, respectively. The parentheses display the probability.